WEEKLY TEST TARGET JEE R & B MATHEMATICS SOLUTION 21 JULY 2019

61. (c) Accordingly, $\frac{T_2}{T_3} = \frac{{}^nC_1 a^{n-1} b}{{}^nC_2 a^{n-2} b^2}$ ......(i)

$\frac{T_3}{T_4} = \frac{{}^{n+3}C_2 a^{n+1} b^2}{{}^{n+3}C_3 a^n b^3}$ ......(ii)

(i) = (ii) $\Rightarrow \frac{2n}{n(n-1)} = \frac{6(n+3)(n+2)}{2(n+3)(n+2)(n+1)}$

$\Rightarrow 2(n+1) = 3(n-1) \Rightarrow n = 5$.

62. (d $\sum_{k=1}^{n} k^3 \left(\frac{C_k}{C_{k-1}}\right)^2 = \sum_{k=1}^{n} k^3 \left(\frac{n-k+1}{k}\right)^2 \left[\because \frac{{}^nC_k}{{}^nC_{k-1}} = \frac{n-k+1}{k}\right]$

$\sum_{k=1}^{n} k(n-k+1)^2 = \sum_{k=1}^{n} k[(n+1)^2 - 2k(n+1) + k^2]$

$= (n+1)^2 \sum_{k=1}^{n} k - 2(n+1)\sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} k^3$

$= (n+1)^2 . \frac{n(n+1)}{2} - 2(n+1). \frac{n(n+1)(2n+1)}{6} + \frac{n^2(n+1)^2}{4}$

$= \frac{n(n+1)^2}{12} [6(n+1) - 4(2n+1) + 3n\ ]$

$= \frac{n(n+1)^2}{12} .(n+2) = \frac{n(n+2)(n+1)^2}{12}$

Trick : Check by taking n = 1, 2.

63. (b) Given $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r \theta$

$\Rightarrow \sin n\theta = b_0 \sin^0 \theta + b_1 \sin^1 \theta$

$+ b_2 \sin^2 \theta + b_3 \sin^3 \theta + ... + b_n \sin^n \theta$

$\Rightarrow \sin n\theta = b_0 + b_1 \sin\theta + b_2 \sin^2 \theta + ... + b_n \sin^n \theta$

(n is an odd integer)

$\because \sin n\theta = {}^nC_1 \sin\theta \cos^{n-1}\theta - {}^nC_3 \sin^3\theta \cos^{n-3}\theta + ...$

$= {}^nC_1 \sin\theta.(1-\sin^2\theta)^{(n-1)/2}$

$- {}^nC_3 \sin^3\theta(1-\sin^2\theta)^{(n-3)/2} + ...$

$\therefore\ b_0 = 0, b_1 =$ coefficient of $\sin\theta = {}^nC_1 = n$

($\because$ $n-1 = n-3$ are all even integers)

64. (a) $(1-x+x^2)^n = a_0 + a_1 x + a_2 x^2 + ... + a_{2n} x^{2n}$

Putting $x = 1$, we get

$(1-1+1)^n = a_0 + a_1 + a_2 + ... + a_{2n}$

$\Rightarrow 1 = a_0 + a_1 + a_2 + ... + a_{2n}$ .....(i)

Putting $x = -1$, we get

$\Rightarrow 3^n = a_0 - a_1 + a_2 - .... + a_{2n}$ ......(ii)

Adding (i) and (ii), we get

$\frac{3^n + 1}{2} = a_0 + a_2 + a_4 + ... + a_{2n}$.

65. (c) $S = 1 + \frac{1}{5} + \frac{1.3}{5.10} + \frac{1.3.5}{5.10.15} + ...$

$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + ..$

$\Rightarrow nx = \frac{1}{5}$ and $\frac{n(n-1)x^2}{2!} = \frac{1.3}{5.10}$

$\Rightarrow n = -\frac{1}{2}$ and $x = \frac{-2}{5}$

$\therefore\ S = \left(1-\frac{2}{5}\right)^{-1/2} = \left(\frac{3}{5}\right)^{-1/2} = \sqrt{\frac{5}{3}}$.

66. (c) We have $\left(1+\frac{C_1}{C_0}\right)\left(1+\frac{C_2}{C_1}\right)....\left(1+\frac{C_n}{C_{n-1}}\right)$

$= \left(1+\frac{n}{1}\right)\left(1+\frac{n(n-1)/2!}{n}\right)...\left(1+\frac{1}{n}\right)$

$= \frac{(1+n)}{1}.\frac{(1+n)}{2}.\frac{(1+n)}{3}....\frac{(1+n)}{n} = \frac{(n+1)^n}{n!}$

67. (a) $\sum_{r=0}^{n}(-1)^r\ {}^nC_r\left(\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+....\text{upto } m \text{ terms}\right)$

$= \sum_{r=0}^{n}(-1)^r\ {}^nC_r.\frac{1}{2^r}+\sum_{r=0}^{n}(-1)^r.{}^nC_r\,\frac{3^r}{2^{2r}}$

$+\ \sum_{r=0}^{n}(-1)^r\ {}^nC_r\,\frac{7^r}{2^{3r}}+.....$

$= \left(1-\frac{1}{2}\right)^n+\left(1-\frac{3}{4}\right)^n+\left(1-\frac{7}{8}\right)^n+....$ up to m terms.

$= \frac{1}{2^n}+\frac{1}{4^n}+\frac{1}{8^n}+....$ upto m terms

$= \frac{\frac{1}{2^n}\left(1-\frac{1}{2^{nm}}\right)}{\left(1-\frac{1}{2^n}\right)} = \frac{2^{mn}-1}{2^{mn}(2^n-1)}$

68. (b) Given $2^n = 1024$, $\therefore\ n = 10$

$\therefore$ The greatest coefficient is ${}^{10}C_5 = 252$.

69. (b) Given expression can be written as

$= \frac{(1+x)^{1/2}+(1-x)^{2/3}}{1+x+(1+x)^{1/2}}$

$\frac{\left[1+\frac{1}{2}x-\frac{1}{8}x^2....\right]+\left[1-\frac{2}{3}x-\frac{1}{9}x^2-....\right]}{1+x+\left[1+\frac{1}{2}x-\frac{1}{8}x^2+....\right]}$

$\frac{\left[1-\frac{1}{12}x-\frac{1}{144}x^2....\right]}{\left[1+\frac{3}{4}x-\frac{1}{16}x^2....\right]} = 1-\frac{5}{6}x+... = 1-\frac{5}{6}x$

when x^2 ,x^3 .... are neglected.

70. (b) If n is odd, then numerically the greatest coefficient in the expansion of $(1-x)^n$ is ${}^nC_{(n-1)/2}$ or ${}^nC_{(n+1)/2}$,

Therefore in case of $(1-x)^{21}$, the numerically greatest coefficient is ${}^{21}C_{10}$ or ${}^{21}C_{11}$.

Therefore the numerically greatest term $= {}^{21}C_{11}x^{11}$ or ${}^{21}C_{10}x^{10}$

$\therefore\ {}^{21}C_{11}x^{11} > {}^{21}C_{12}x^{12}$ and ${}^{21}C_{10}x^{10} > {}^{21}C_9x^9$

$\Rightarrow\ \frac{21!}{10!\,11!} > \frac{21!}{9!\,12!}x$ and $\frac{21!}{11!\,10!}x > \frac{21!}{9!\,12!}$

$\Rightarrow\ \frac{6}{5} > x$ and $x < \frac{5}{6} \Rightarrow x \in \left(\frac{5}{6},\frac{6}{5}\right)$

71. (a,c) Since coefficients ${}^mC_1$, ${}^mC_2$ and ${}^mC_3$ of T_2, T_3, T_4 *i.e.* are the first, third and fifth terms of an A. P., which will also be in A. P. of common difference $2d$.

Hence $2\,{}^mC_2 = {}^mC_1 + {}^mC_3 \Rightarrow (m-2)(m-7) = 0$. Since 6th term is 21, m = 2 is ruled out and we have m = 7 and $T_6 = 21 = {}^7C_5\left[\sqrt{2^{\log(10-3^x)}}\right]^{7-5}\times\left[\sqrt[5]{2^{(x-2)}\log 3}\right]^5$

$\Rightarrow\ 21 = 21.\,2^{\log(10-3^x)+\log 3^{x-2}}$

$\Rightarrow\ 2^{\log[(10-3^x)\,3^{x-2}]} = 1 = 2^0$

Which on simplification gives $x = 0, 2$.

72. (c,d) We have $\left[2^{\log_2\sqrt{9^{x-1}+7}} + \frac{1}{2^{(1/5)\log_2(3^{x-1}+1)}}\right]^7$

$= \left[\sqrt{9^{x-1}+7} + \frac{1}{(3^{x-1}+1)^{1/5}}\right]^7$

$\therefore\ T_6 = {}^7C_5\left(\sqrt{9^{x-1}+7}\right)^{7-5}\left[\frac{1}{(3^{x-1}+1)^{1/5}}\right]^5$

$= {}^7C_5(9^{x-1}+7)\frac{1}{(3^{x-1}+1)}$

Now $T_6 = 84 \Rightarrow {}^7C_5\frac{(9^{x-1}+7)}{(3^{x-1}+1)} = 84$

$\Rightarrow\ 9^{x-1}+7 = 4(3^{x-1}+1)$

$\Rightarrow\ 3^{2x} - 12(3^x) + 27 = 0 \Rightarrow y^2 - 12y + 27 = 0$

(Where $y = 3^x$)

$\Rightarrow\ y = 3,9 \Rightarrow 3^x = 3,9 \Rightarrow x = 1,2$

73. (b) $(1+2x)^{-1/2}$ can be expanded if $|2x| < 1$ *i.e.* if $|x| < \frac{1}{2}$, *i.e.* if $-\frac{1}{2} < x < \frac{1}{2}$ *i.e.* if $x \in \left(-\frac{1}{2}, \frac{1}{2}\right)$.

74. (c) $49^n + 16n - 1 = (1+48)^n + 16n - 1$

$1 + {}^nC_1(48) + {}^nC_2(48)^2 + ... + {}^nC_n(48)^n + 16n - 1$

$= (48n + 16n) + {}^nC_2(48)^2 + {}^nC_3(48)^3 + ... + {}^nC_n(48)^n$

$= 64n + 8^2[{}^nC_2.6^2 + {}^nC_3.6^3.8 + {}^nC_4.6^4.8^2 + .. + {}^nC_n.6^n.8^{n-2}]$

Hence, $49^n + 16n - 1$ is divisible by 64.

75. (c) Middle term in expansion of $(1+\alpha x^4) = {}^4C_2(\alpha x)^2$

Middle term in expansion of $(1-\alpha x)^6 = {}^6C_3(-\alpha x)^3$

According to question, ${}^4C_2\alpha^2 = -\ {}^6C_3\alpha^3$

$\Rightarrow\ \alpha = -3/10$.

76. (c) In the expansion of $(1+x)^{2n}$, the general term

$= {}^{2n}C_k, 0 \le k \le 2n$

As given for $r > 1, n > 2, {}^{2n}C_{3r} = {}^{2n}C_{r+2}$

$\Rightarrow$ Either $3r = r + 2$

or $3r = 2n - (r+2)$, $\quad (\because {}^nC_r = {}^nC_{n-r})$

$\Rightarrow r = 1$ or $n = 2r + 1 \Rightarrow n = 2r + 1$, $\quad (\because r > 1)$.

77. (a) In the expansion of $(1+x)^n$, it is given that ${}^nC_1, {}^nC_2, {}^nC_3$ are in A.P.

$\Rightarrow 2.{}^nC_2 = {}^nC_1 + {}^nC_3$

$\Rightarrow 2.\frac{n(n-1)}{1.2} = \frac{n}{1} + \frac{n(n-1)(n-2)}{1.2.3}$

$\Rightarrow 6(n-1) = 6 + (n-2)(n-1)$

$\Rightarrow n^2 - 9n + 14 = 0 \Rightarrow n = 2$ or $n = 7$.

But $n = 2$ is not acceptable because, when $n=2$, there are only three terms in the expansion of $(1+x)^2$, $\therefore\ n = 7$.

78. (b) $T_{r+1} = {}^nC_r(a)^{n-r}(-b)^r$.

$T_5 = T_{4+1} = {}^nC_4a^{n-4}(-b)^4 = {}^nC_4a^{n-4}b^4$

and 6^{th} term

$= (T_6) = T_{5+1} = {}^nC_5\,a^{n-5}(-b)^5 = -{}^nC_5\,a^{n-5}\,b^5$

Since $T_5 + T_6 = 0$, therefore

$${}^nC_4\,a^{n-4}\,b^4 - {}^nC_5\,a^{n-5}\,b^5 = 0 \Rightarrow \frac{a^{n-4}\,b^4}{a^{n-5}\,b^5} = \frac{{}^nC_5}{{}^nC_4}$$

$$\Rightarrow \frac{a}{b} = \frac{n!}{(n-5)!\;5!}\cdot\frac{4!\;(n-4)!}{n!} \Rightarrow \frac{a}{b} = \frac{n-4}{5}.$$

79. (a) T_3, T_4, T_5 in the given expansion are respectively ${}^{10}C_2\,2^8\left(\frac{3x}{8}\right)^2,\ {}^{10}C_3\,2^7\left(\frac{3x}{8}\right)^3,\ {}^{10}C_4\,2^6\left(\frac{3x}{8}\right)^4$

or $1620x^2, 810x^3, \frac{8505}{32}x^4$

We are given that T_4 is numerically the greatest term so that $|T_4| > |T_3|$ and $|T_4| > |T_5|$

$\therefore\ |x| > 2$ and $\frac{64}{21} > |x|$

$2 < |x| < \frac{64}{21}$ .....(i)

The above inequality (i) is equivalent to two inequalities $2 < x < \frac{64}{21}$ and $-\frac{64}{21} < x < -2$

80. (c) Required probability is

$P(\text{getting } 8) + P(9) + P(10) + P(11) + P(12)$

$= \frac{5}{36} + \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{15}{36} = \frac{5}{12}.$

81. (d) The chance of head $= \frac{1}{2}$ and not of head $= \frac{1}{2}$

Since A has first throw, he can win in the first, third, …

$\therefore$ Probability of A's winning

$= \frac{1}{2} + \left(\frac{1}{2}\right)^2 \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^4 \cdot \frac{1}{2} + ...$

$= \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + ... = \frac{2}{3}.$

82. c) Since there are one A, two I and one O, hence the required probability $= \frac{1+2+1}{11} = \frac{4}{11}.$

83. (b) Required probability is $1 - P$ (All letters in right envelope) $= 1 - \frac{1}{n!}$

{As there are total number of $n!$ ways in which letters can take envelopes and just one way in which they have corresponding envelopes}.

84. (a) Favourable ways $\{29, 92, 38, 83, 47, 74, 56, 65\}$

Hence required probability $= \frac{8}{100} = \frac{2}{25}.$

85. (c) Total rusted items $= 3 + 5 = 8$; unrusted nails $= 3$.

$\therefore$ Required probability $= \frac{3+8}{6+10} = \frac{11}{16}.$

86. (b) It is obvious.

87. (b) Here $P(A) = 0.4$ and $P(\overline{A}) = 0.6$

Probability that A does not happen at all $= (0.6)^3$

Thus required Probability $= 1 - (0.6)^3 = 0.784$.

88. (a) $P(A' \cap B') = \frac{4}{5} \cdot \frac{3}{4} = \frac{3}{5}.$

89. (b) Required probability is $1 - P(\text{no girl}) = 1 - \left(\frac{1}{2}\right)^4 = \frac{15}{16}.$

90. (b) A determinant of order 2 is of the form $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

It is equal to $ad - bc$. The total number of ways of choosing a, b, c and d is $2 \times 2 \times 2 \times 2 = 16$. Now $\Delta \neq 0$ if and only if either $ad = 1,\ bc = 0$ or $ad = 0,\ bc = 1$. But $ad = 1,\ bc = 0$ iff $a = d = 1$ and one of b, c is zero. Therefore $ad = 1,\ bc = 0$ in three cases, similarly $ad = 0,\ bc = 1$ in three cases. Therefore the required probability $= \frac{6}{16} = \frac{3}{8}.$